1. $\lim_{x \to -6^+} \frac{12x + 72}{x^2 - 36}$

Applying direct substitution

$\lim_{x \to -6^+} = \frac{12(-6) + 72}{-6^2 - 36} = \frac{0}{0}$

Applying L. Hopitals rule where $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f'(x)}{g'(x)}$

$\lim x \to -6^+ \quad \frac{12x + 72}{x^2 - 36} = \left| \frac{12}{2x} \right|_{-6} = \frac{12}{2(-6)} = -1$

2. $P = 8000$

$A = 15000$

$t = 90$ months $= \frac{90}{12} = 7.5$ years

$A = Pe^{rt}$

$\frac{A}{P} = e^{rt}$ where t is time in year

$\frac{15000}{8000} = e^{7.5r}$

$1.875 = e^{7.5r}$

$7.5r = \ln 1.875$

$\frac{7.5r}{\cancel{7.5}} = \frac{0.6286}{7.5}$

$r = 0.0838 = 8.38\%$ per year

3. $f(x) = -100x^{1.25} - 10x^{-0.7} + 8250$

Using nth rule

$f'(x) = -125x^{0.25} + 7x^{-1.7}$

4. $f(x) = \ln(7.5x^2 - 11x + 37.5)$

$f'(x)$ of $\ln f(x) = \frac{f'(x)}{f(x)} = \frac{15x - 11}{7.5x^2 - 11x + 37.5}$

5. $f(t) = e^{(-1.4t^3 + \ln 2.45 + 6.98t)}$

$f'(t)$ of $e^{f(t)} = f'(t)\, e^{f(t)}$

$= [-4.2t^2 + 6.98]\, e^{(-1.4t^3 + \ln 2.45 + 6.98t)}$

6. $y = f(x) = (2.085t^2 - 72.5t + 2.75e)^{1.625}$

Let $(2.085t^2 - 72.5t + 2.75e) = u$.

$y = u^{1.625}$

$\frac{dy}{du} = 1.625\, u^{0.625}$

$\frac{du}{dt} = 4.17t - 72.5$

By chain Rule

$\therefore \frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt}$

$= 1.625\, u^{0.625} \cdot 4.17t - 72.5$

$= 1.625 [2.085t^2 - 72.5 + 2.75e]^{0.625} [4.17t - 72.5]$

7. $f(x) = (x^{2.4} - 2.6 \ln e)(10x^{1.3} - e^{-x})$

$\ln e = 1 \quad \therefore \quad 2.6 \ln e = 2.6$

$f(x) = (x^{2.4} - 2.6)(10x^{1.3} - e^{-x})$

~~Expanding the bracket~~

Using product rule

$f'(x) = f'g + g'f$

$= (2.4x^{1.4})(10x^{1.3} - e^{-x}) + (13x^{0.3} + e^{-x})(x^{2.4} - 2.6)$

$= 24x^{2.7} - e^{-x} + 13x^{2.7} + 33.8x^{0.3} + x^{2.4}e^{-x} - 2.6e^{-x}$

8. $f(w) = 13^{1.6w^3 - 3.2w}$

$f'(w) = 13 \ln e^{1.6w^3 - 3.2w}$

$f'(w) = \ln(13)(4.8w^2 - 3.2) \cdot 13^{\frac{8}{5}w^3 - \frac{16}{5}w}$

9. $f(x) = \dfrac{1.74e^{x} - 4e^{-x}}{12.98x^{2} + e^{1.06}}$

Using quotient rule where $f'(x) = \dfrac{f'g - g'f}{g^{2}}$

$f'(x) = \dfrac{1.74e^{x} + 4e^{-x} - 25.96x}{(12.98x^{2} + e^{1.06})2}$

10. $f(x) = \log_{6}(5x^{2.5} - 7x)$.

$\frac{d}{dx} \log_{a} f(x) = \dfrac{1}{f(x)\ln a} f'(x)$

$= \dfrac{1}{(5x^{2.5} - 7.5)\ln 6}(12.5x^{1.5} - 7)$

$= \dfrac{12.5x^{1.5} - 7}{(5x^{2.5} - 7.5)\ln 6}$

11. $y = 5\ln w + \ln 4 + w^{1/4}$.

$\frac{dy}{dw} = \frac{5}{w} + \frac{1}{4}w^{-3/4}$.

$= \frac{1}{5}w + \frac{1}{4\cancel{w}^{3/4}}$.

12. $\int(40x^{1.5} - 12x^{0.5} + e^{x})\,dx$ Using nth rule.

$\int 40x^{1.5} + e^{x} - 12x^{0.5}\,dx = \dfrac{40x^{2.5}}{2.5} + e^{x} + c - \dfrac{12x^{1.5}}{1.5}$

$= 16x^{2.5} + e^{x} - 8x^{1.5}$

$= 16x^{1.5} \times 16x + e^{x} - 8x^{1.5}$

$= 8x^{1.5} + 16x + e^{x}$

13. $\int (4.8x^{-5} + 14x - \frac{3}{x})\, dx$

N^{th} rule

$\int (4.8x^{-5} + 14x - 3/x)\, dx = \frac{4.8x^{-4}}{-4} + \frac{14x^2}{2} - 3\ln x$

$= -1.2x^{-4} + 7x^2 - 3\ln(x)$

14. $x = 36 \ln 540 - 48 \ln P$

a) Elasticity of demand $= \frac{dx}{dp} \cdot \frac{P}{\cancel{x}}$

$\frac{dx}{dp} = \left[36(0) - \frac{48}{P}\right] \frac{P}{\cancel{x}}$

$\frac{dx}{dp} = \left[-\frac{48}{\cancel{P}}\right] \frac{\cancel{P}}{x} = \frac{-48}{x} = -\frac{48}{P}$

b) $S.P = 250$

$\frac{dx}{dp} = -\frac{48}{P} = -\frac{48}{250} = -0.1920$

c) $S.P = 400$

$\frac{dx}{dp} = -\frac{48}{400} = -0.1200$

d) $S.p = 304.50$

$\frac{dx}{dp} = -\frac{48}{P} = \frac{-48}{304.50} = 0.1576$

15. $P = 60x^{-1/2}$.

$c(x) = 0.40x + 800$

a) Revenue = Selling price − Costs

$= 60x^{-1/2} - 0.40x - 800$

b) Marginal revenue = $\dfrac{\text{change in total Revenue}}{\text{change in qty sold}}$

If one extra key is sold

$M.R = \dfrac{60x^{-1/2} - 0.40x - 800}{1}$

b) Revenue = profit = $60x^{-1/2} - 0.40x - 800$

d) ~~total~~ incremental revenue = Price * No of units

$= 60x^{-1/2} * 1 = 60x^{-1/2} = \dfrac{60}{\sqrt{3250}} = 1.052$

e) Marginal profit = Marginal revenue / units sold

$= (60x^{-1/2} - 0.40x - 800)/1$

(f) $x = 6725$

Incremental profit = Profit of 6725 & 6724.

When $x = 6725$ Profit $= 60x^{-1/2} - 0.40x - 800$ $= 1430.2658 = -3489.2883$

$x = 6724$ Profit $= 60(6724)^{-1/2} - 0.40(6724) - 800 = -3488.8682$

Incremental profit = -0.4001

16. $p(x) = 198x^{1/2} - 8x - 625$

Rate of Δ of

a) profit $= \frac{dp}{dx} = 99x^{-\frac{1}{2}} - 8$

For maximum profit $\frac{dp}{dx} = 0$

$$\frac{99}{x^2} - 8 = 0$$

$$\frac{99}{8} = \frac{\cancel{8}x^2}{\cancel{8}}$$

$$x = 3.5 \simeq 4$$

2nd derivative test

$\frac{d^2p}{dx^2} = -\frac{99}{2}x^{-3/2}$ replacing x with 4,

$\frac{d^2p}{dx^2} = -\frac{99}{2}(4)^{-3/2} = -6.1875$ this negative

figure shows that the value of x plugged in gives maximum rate of change which is the profit

b) Total weakly profit $= \cancel{99(x)^{-1/2} - 8}$

$$= 198x^{1/2} - 8x - 625$$

$$= 198(4)^{1/2} - 8(4) - 625$$

$$= -261$$